EXTENSIONS OF *-REPRESENTABLE POSITIVE LINEAR FUNCTIONALS TO UNITIZED QUASI *-ALGEBRAS

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The possibility of a GNS construction for a positive linear functional defined on a *-algebra with unit, was proved by Powers in 1971. If the *-algebra $\mathfrak{A}$ has no unit, a positive linear functional can be extended to a positive linear functional ω^e on its unitization $\mathfrak{A}^e = \mathfrak{A} \oplus \mathbb{C}$ if, and only if, it is *hermitian* i.e. $\omega(a^*) = \overline{\omega(a)}$, $\forall a \in \mathfrak{A}$ and *Hilbert bounded*, i.e. there exists $\delta > 0$ such that $|\omega(a)|^2 \leq \delta\omega(a^*a)$, for every $a \in \mathfrak{A}$.
Contrarily to the case of *-algebras, a positive linear functional defined on *quasi* *-algebra $(\mathfrak{A}, \mathfrak{A}_0)$ with unit is *not* automatically *-representable. For this reason, if $(\mathfrak{A}, \mathfrak{A}_0)$ has no unit, the notion of *extensibility* has to be modified: a positive linear functional ω will be called *extensible* if it is the restriction to $(\mathfrak{A} \oplus \{0\}, \mathfrak{A}_0 \oplus \{0\})$ of some **-representable* positive linear functional ω^e defined on the unitization $(\mathfrak{A}^e, \mathfrak{A}_0^e)$.
Here, starting from a hermitian linear functional ω defined on a quasi *-algebra $(\mathfrak{A}, \mathfrak{A}_0)$ without unit, we study under what conditions it is possible to extend ω to a *-representable linear functional, defined on a quasi *-algebra with unit, possibly the whole unitization of $(\mathfrak{A}, \mathfrak{A}_0)$. We give a new condition on ω (precisely, we ask that there exists $\zeta > 0$ such that $|\omega(x)| \leq \zeta\{\sup_{a\in\mathfrak{A}_0,\omega(a^*a)=1} |\omega(x^*a)|^2\}^{1/2}$, for every $x \in \mathfrak{A}$) to make the extension ω^e to the unitized quasi *-algebra $(\mathfrak{A}^e, \mathfrak{A}_0^e)$ *-representable; this new condition is quite natural, in fact we prove that it reduces to Hilbert boundedness on $\mathfrak{A}_0$; moreover if $(\mathfrak{A}, \mathfrak{A}_0)$ has a unit, then this condition is automatically fulfilled, hence, under this condition, *-representability of ω and its extensibility are equivalent.